There have been numerous neurobiological theories positing that quantum mechanics is importantly responsible for certain psychological phenomena, such as consciousness and conscious decision-making. In this respect, such theories understand the relevant aspects of the brain to be “quantum mechanical.” Although it is highly questionable whether any of these theories have the requisite or sufficient empirical evidence strictly within the domain of neuroscience to appropriately justify their views, a different approach to arguing that quantum mechanics is importantly relevant to human decision-making will be discussed. By putting various puzzle pieces together of empirical evidence in psychology that quantum probability models describe certain human cognitive decision-making processes, I contend that human decision-making overall is best described by quantum mechanical processes. Finally, I at times provide a novel defense of this view from a number of objections.

**Keywords:** Decision-making, quantum cognition, quantum mechanics, cognitive processes, mathematical models, classical cognition
1. Quantum Mechanical Theories in Neurobiology

As is well documented in the literature, quantum effects have been shown to percolate up at levels higher than the subatomic and atomic domains. For instance, quantum mechanics can account for the production of laser light. It also provides the foundation for all of chemistry. Furthermore, it is found to be at work for solids and fluids. The special properties of silicon and silicon chips are directly accounted for by the existence of electron waves in the relevant solids (Cullis & Canham 1991). In fact, quantum behavior is responsible for why some solids are insulators, others are metals that conduct electricity, and others are semiconductors. The fact that some metals become superconductors completely devoid of electrical resistance and that some fluids become superfluids that can flow over surfaces without friction is also a manifestation of quantum behavior (Greiner et al. 2002). Moreover, the thermal properties of a solid, such as its heat, depend upon the quantum motion of the atoms that constitute the solid. There is also the burgeoning new field of quantum computers, where the computer’s “mind” and software supervenes upon and is explained by quantum physical states.

Furthermore, quantum behavior has been shown to be responsible for macro effects at the biological level. For example, quantum coherence plays a role in photosynthesis (Romero et al 2014). Moreover, biologists widely maintain that quantum uncertainty at the level of DNA mutations percolate up to have significant population effects. Jacqueline Barton has experimentally shown that electron quantum tunneling, where electrons go through barriers that it could not on a classical picture, in DNA occurs across as many as sixty base-pairs, which at times causes damage, lesions, and repairs to DNA (Barton 1999). Such damage, lesions, and repairs are believed to have an influence on mutations, evolution, and the development of cancer.

As quantum behavior can have effects on chemical bonding, solids, liquids, DNA, and the evolution of species including our own, perhaps it may be the case that quantum effects play a role in the brain and mind in cognitive decision-making. Given the influence of quantum mechanics at the above various levels, the idea that it influences decision-making may not be so far-fetched.
There have been several theorists attempting to describe certain psychological phenomena with quantum mechanics at the neurobiological rather than psychological level (Lockwood 1989, Penrose 1989, 1994, Beck and Eccles 1992, Stapp 1993, Hameroff and Penrose 1996, Vitiello 2001, Smith 2003, Tse 2013). However, such work is highly controversial, where there certainly is no consensus agreement for the claim that an examination of quantum neuroscience demonstrates that such phenomena can be appropriately described by quantum mechanics (Tegmark 2000).¹ This is still a highly contentious subject matter. There are various general common criticisms against these views, such as that they do not have sufficient empirical support and that such views run into the problem of decoherence. As objections against neurobiological theorists who attempt to show that decision-making is quantum mechanical is well-documented in the literature, I will not go into further detail here. Nevertheless, I will not be taking a stand on the quantum neurobiological claim regarding decision-making. Rather, regardless of whether neuroscientists who espouse a quantum mechanical view are correct or incorrect, I focus and base my case that a quantum formalism applies to decision-making processes based on experimental psychological data. My emphasis will be on the psychology rather than the neurobiology of decision-making. To note, while quantum theories generally may attempt to account for various psychological phenomena, due to obvious space concerns, the focus of our discussion will only be on cognitive decision-making and not on qualia or any other aspect of the mind.

2. Quantum Mechanical Cognitive Decision-Making

We will now discuss robust and replicated empirical evidence of quantum effects at the psychological level in decision-making (Busemeyer & Bruza 2012, Bruza et al. 2015). By putting various puzzle pieces together in the psychological literature, I will conclude that decision-making at the psychological level can best be described by quantum mechanical models.

¹ For a response to Tegmark’s criticism of Hameroff and Penrose’s model, see Hagan et al. 2002.
Various quantum cognitive scientists have run psychological experiments demonstrating that disparate aspects of decision-making can be best described with a quantum model. While in their works they primarily narrowly focus on particular kinds of decision-making, I will put together and synthesize these different studies in order to step back and provide an overall big picture view in regards to decision-making, generally understood. I will contribute to the literature by synthesizing the disparate studies and arguing for the position that psychological decision-making is in fact best described by a quantum formalism. Furthermore, I at times will provide new defenses of this view from several objections.

To note, the quantum models that I will discuss below will be able to appropriately describe various decision-making phenomena that generally cause difficulties for classical probability theory. The focus of this paper specifically will be on how quantum psychological theories of decision-making are able to best describe the relevant phenomena, although some brief points regarding explanatory adequacy will now be discussed. The general distinction between descriptive and explanatory adequacy can be drawn from Chomsky (1995). As applied to decision-making, a quantum theory of decision-making is descriptively adequate if it provides an appropriate description of the computational processes that underwrite decision-making. However, a quantum view is explanatorily adequate if it can provide a foundational principled justification as to why a quantum approach to decision-making is appropriate such that accurate predictions may be made. While quantum models may be descriptively adequate, further work needs to be undertaken to demonstrate that they are explanatorily adequate. One may list facts describing the computational processes that underwrite decision-making, but understanding such facts is another matter. Blutner and beim Graben point out that quantum models do not automatically provide a true explanation of such phenomena since

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2 To note, all of the relevant studies contain detailed and very long mathematical models for the psychological phenomena, many of which take up an entire article. While I do provide a very basic and general mathematical framework of quantum cognitive models for decision-making below, for space concerns, I do not slavishly regurgitate the complete mathematics of the various models here, but invite those interested in the more precise details concerning the mathematics to view them in the cited references.
normally, such models introduce new parameters, such as size and sign of the interference effect, which can be fitted to empirical data without necessarily explaining them (2015). However, since in most cases we do not know the values of the new parameters, predictions are not possible. Although quantum models can appropriately describe the phenomena, explanations in quantum cognition are less frequently found. Hence, deeper foundational issues must be addressed in providing independently motivated first principles as to why quantum mechanical theories of cognition are appropriate in order for quantum models to provide a deeper understanding and true explanation of the phenomena. To note, there are exceptions to this, where certain quantum models in specific cases do explain and predict. For instance, Wang and Busemeyer have shown their quantum model makes a precise a priori explanation and prediction of a particular pattern of order effects known as the QQ (quantum question) equality (Wang & Busemeyer 2013, Wang et al. 2014). Their predictions have been confirmed over non-quantum models, such as the anchoring-and-adjustment model, based on seventy experiments.

There are several arguments for the necessity of quantum models of decision-making such that these models do have explanatory adequacy. For example, Busemeyer and Bruza provide a number of such contentions (2012). One is that the order of questions matters at many times in decision-making. For instance, a love-starved teenager will give a different answer to question B, “How happy are you?”, if this question is preceded by question A, “When was the last time you had a date?” As A and B are two measurements, where the probabilities of the outcomes are contingent upon the ordering of the questions, the measurements for A and B are non-commutative. Classical probability theory obeys the commutative axiom, while quantum probability theory can violate it and thus, can explain the phenomena. Also, they state that judgements do not always obey classic logic. For instance, they write that the distributive axiom is frequently violated in decision-making, which goes against classic logic. However, quantum logic does not obey the distributive axiom, and thus, quantum models can explain such phenomena. Also, from the perspective of “operational realism” and bounded rationality, Blutner and beim Graben point out that an agent with limited mental resources is not able to
maintain all possible preference rankings and perspectives (2015). Thus, he or she has to choose one particular perspective upon a given decision problem. Such perspectives are complementary to each other, which lead to different Hilbert subspaces in a projective lattice. They propose that this is a foundational motivation for quantum theories of cognition. As the focus of this paper is primarily on the descriptive adequacy of quantum models of decision-making over classical probability models, our focus largely now will be on this topic, although some of the below studies I will discuss at times also may have explanatory adequacy.

Quantum mechanics was invented by physicists in order to describe phenomena that were indescribable from a classical vantage point, amongst other reasons. Likewise, some cognitive scientists and physicists have used quantum mechanics in order to describe psychological phenomena that elude classical decision-making models in psychology. I take it that most philosophers and perhaps even many cognitive scientists are not fully aware or do not see the full import of quantum cognitive models for decision-making. The descriptive adequacy of Newtonian physics is very similar to the descriptive adequacy of Einstein’s general theory of relativity in most situations with only small differences. However, confirmation of the general theory of relativity’s ability to better describe phenomena over Newton’s natural philosophy was made based primarily on only three different kinds of phenomena: changes in the perihelion of Mercury, gravitational redshifts, and the bending of light rays around the sun. Likewise, what we will find below concerning the classical versus quantum models of cognitive processing theories for decision-making is that they are able to at times both describe phenomena, especially for ordinary cases of decision-making, but there are differences, where classical probability theory fails to appropriately describe certain psychological fallacies and heuristics. On a number of counts, empirical evidence confirms the descriptive adequacy of quantum theories as opposed to classical probability theories for such cases, which allows us to conclude that quantum theories of cognitive processing for decision-making are superior to classical-based theories in this regard.

Von Neumann is responsible for providing an axiomatic foundation to quantum mechanics, and in so doing, he found that it implied a new type of probability theory (Neumann 1932). Currently, the two general theories
for assigning probabilities to events are the classical (Kolmogorov 1933) and the quantum approaches. Classical probability theory defines events as subsets of a universal set, and it obeys all the laws of Boolean algebra. However, due to a Hilbert space representation, where events are subspaces of a Hilbert space, quantum probability theory follows all the laws of Boolean algebra but not orthomodularity and the commutative axiom. Because of this fact, quantum probability theory does not have to obey the law of total probability, which is a fundamental rule of classical probability theory that provides the total probability of a particular outcome that can be realized by way of several different events. However, the classical theory does have to obey the law of total probability. Let us understand $A$ as being any event in a sample space $S$. If $S$ is partitioned by the disjoint events $B_1, B_2, B_3, \ldots, B_k$ such that $Pr(B_i) > 0$ for $i = 1, 2, 3, \ldots, k$, then the law of total probability states:

$$Pr(A) = Pr(A|B_1) \cdot Pr(B_1) + Pr(A|B_2) \cdot Pr(B_2) + \ldots + Pr(A|B_k) \cdot Pr(B_k)$$

The take-home message here is that there are significant and tractable differences between these two probability theories, and it is in significant part an empirical question as to which general theory is more successful in describing psychological decision-making phenomena. One method to discern which probability theory is at work in human decision-making is to see whether or not the law of total probability is violated and whether quantum models instead can account for the phenomena. While in the below studies, experiments demonstrate the ability of quantum mechanical models of decision-making to account for the psychological phenomena that cause difficulties for classical probability models, it is the general reliance on and use of differences between both probability theories that allows our theorists and experimenters to differentiate and adjudicate between the two. What is interesting is that quantum theory was initially created to describe

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3 To note, not every logic that has similarities with a Boolean algebra but does not obey distributivity is a basis for quantum probability theory.

4 To note, in standard quantum logic, commutativity is satisfied. However, when we consider the properties of projection operators it is true that they do not commute in many cases. Moreover, we find the puzzling fact that the composition of two projectors is not even a projector if the two projectors do not commute.
non-commutative findings in physics that seemed paradoxical from the viewpoint of classical physics, *inter alia*. Now, in cognitive science, non-commutative findings such as order effects are being successfully described by models that are based on quantum probability theory.

According to a model of cognitive processing for decision-making that is based on classical probability theory, such as the Markov model, an individual only has one precise preference order at a specific moment in time, although the order may change across time. For instance, at moment $t_1$, an agent prefers action A over B and B over C. However, the process can move from one basis state to another during deliberation. In other words, the preference order can change over time. A final decision is an end result of a sequence of transitions that constitute a single trajectory from the beginning to the end state. Here, a single trajectory is a deterministic function of time, where the agent is in a determinant basis state immediately before a decision is made and at the end of deliberation. The classical Markov model can give a probability distribution over all the possible trajectories, but for each given decision, there is only one trajectory and no possibility for trajectories to interfere with each other.

However, for a generally understood quantum cognitive process for decision-making, the individual is in a superposition of all the possible preference orders, and the agent is uncommitted to any particular order. In other words, the agent is not in any particular basis state during deliberation. Rather, the agent is in a superposition where all the different basis states or preference orders coexist in parallel, and this indeterminate fuzzy state flows across time like a traveling wave until a decision is made. All of the indeterminate trajectories coexist in parallel across time, and they make a wave of potentialities over the states across time. The agent is not in a determinant basis state immediately before the decision is made. However, it is the self-observed decision that creates a clear and determinant basis state. The agent’s self-observed arrival upon a specific preference order can be described as the collapse of the wave function. The quantum model derives the probability amplitudes for all of the possible trajectories, where all the indeterminate trajectories can interfere with each other. As we can see, the quantum model views cognitive processing as like a wave, where it obeys the laws of wave motion.
To provide a very basic quantum model of cognitive processing for decision-making with a 2-dimensional vector space, let us say that there is some dichotomous question $A$ with only the two possible answers of $a_0 = \text{no}$ and $a_1 = \text{yes}$. For these two possible answers, there is the corresponding vectors of $|a_0\rangle$ and $|a_1\rangle$, respectively, that form a basis of the vector space. For the different dichotomous question $B$, there are the vectors $|b_0\rangle$ and $|b_1\rangle$ corresponding to the answers $b_0 = \text{no}$ and $b_1 = \text{yes}$ that form a certain basis of the same two-dimensional vector space. In this qubit-based quantum formalism, we can use a generic vector $|s\rangle$ called an opinion state to describe a subject’s mental state. The opinion state represents the opinions, reasons, and beliefs of the agent in judgment situations. This state can be simply written with $s_0$ and $s_1$ complex numbers as:

$$|s\rangle = s_0|a_0\rangle + s_1|a_1\rangle$$

Here, we can state that $|s\rangle$ is a superposition of the basis vectors $|a_i\rangle$ ($i = 0, 1$), and $s_i$ are the superposition coefficients. The square modulus $|s_i|^2$ is the subjective probability $Pr(a_i)$ given the opinion state $|s\rangle$ that the answer of question $A$ has the answer $i = 0, 1$. The complex coefficients $s_i$ of $|s\rangle = s_0|a_0\rangle + s_1|a_1\rangle$ can be defined in terms of the inner product $s_i = \langle a_i | s \rangle$ of the vector $|s\rangle$ and the basis states $|a_i\rangle$ ($i = 0, 1$). Therefore, the estimated probability relevant to events $a_i$ given opinion state $|s\rangle$ is:

$$Pr(a_i) = |\langle a_i | s \rangle|^2, \; i = 0, 1$$

Speaking more generally, the inner product $\langle s | s' \rangle$ of two vectors $|s\rangle$ and $|s'\rangle$ can be defined as: if $s_0$ and $s_1$ are the components of $|s\rangle$ in a basis $|a_0\rangle$ and $|a_1\rangle$ and $s'_0$ and $s'_1$ are the components of $|s'\rangle$ with the same basis, then the inner product is $\langle s | s' \rangle = s_0 s'_0* + s_1 s'_1*$. Therefore, the inner product of $|s\rangle$ and itself is $\langle s | s \rangle = |s_0|^2 + |s_1|^2$, and if the vector is normalized, then it is equal to 1. Since the 2-dimensional vector space now has the inner product, it is a Hilbert space, abbreviated as $H$.

For the first kind of quantum cognition study, Busemeyer, Wang, and Lambert-Mogiliansky demonstrate that a quantum probability model can account for cases of categorization and decision-making that are problematic.

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5 I would like to express gratitude to Dan Linford and Paul Park for their help with the mathematics.
for classical probability theory (2009). They compare their quantum model to the Markov theory, which is a general classical mathematical framework common to cognitive science that describes probabilistic-dynamical systems. While the Markov model is derived from the Kolmogorov differential equation, the dynamics for Busemeyer and company’s quantum model are based on the Schrödinger differential equation. The Markov and quantum models use the same type of parameters determined by the utility of actions and/or the strength of prior evidence, but the Markov model includes these parameters into the intensity matrix of the Kolmogorov equation. On the other hand, the quantum model introduces the parameters into the Hamiltonian matrix of the Schrödinger differential equation. Although both models derive the final choice probabilities from the states that came from deliberation, the Markov model remains a wholly linear process. Meanwhile, the quantum model is linear on amplitudes but the response probabilities are determined by their squared magnitudes. By being determined by their squared magnitudes, this leads to an important non-linearity in the process where quantum interference effects at the psychological level can importantly arise. It will be such interference effects in psychological processes that largely will allow for quantum models of decision-making to account for the psychological phenomena that create difficulties for classical probability theory.

The experimenters gave participants pictures of bald, Caucasian, expressionless, male faces that varied along the two dimensions of face width and lip thickness. There were two different distributions of faces. The ‘narrow’ face distribution had a narrow face with thick lips, while the ‘wide’ face distribution had a wide face with thin lips. Half of the faces presented were part of the ‘narrow’ distribution while the other half were part of the ‘wide’ distribution. Participants were asked to categorize faces into a ‘good guy’ and ‘bad guy’ group. Moreover, subjects were asked to decide whether to take an ‘attack’ or ‘friendly’ action to the faces. They were told that they were on the planet Meboo and had to categorize and make decisions on the inhabitants there. ‘Adoks’ tend to have round faces with thin lips and tend to be friendly while ‘Lorks’ tend to have long faces with thick lips and tend to be hostile. However, the above tendencies of the Adoks and the Lorks are not absolute. As in any culture, there is some crossover. Half
of the participants were given the C-then-D condition, where they made a
categorization then an action decision. In the D-then-C condition, the other
half of the subjects made an action decision followed by a categorization
on the same set of pictures. After the completion of the respective tasks, the
participants who were provided with the D-then-C condition were given
the C-alone condition, where participants only made a categorization, and
the other subjects who were provided the C-then-D condition received the
D-alone condition, where subjects made only an action decision. In the
C-alone and D-alone conditions, the same set of pictures was used as the set
of pictures used in the C-then-D and D-then-C conditions. At times, payoffs
were provided to participants. For example, if one was friendly to a friendly
Adok, then the Adok was described as giving you money for being friendly.

Our experimenters found that the law of total probability was violated
and that the Markov model fails. For example, no convex combination or
weighted average of Pr(A|G) and Pr(A|B) from the C-then-D condition can
equal Pr(A) observed in the D-alone condition. For, the latter probability
exceeds both of the former, whereas a convex combination must lie in
between the two relevant probabilities. When a face was categorized as ‘good
(G),’ it was then attacked (A) forty-three percent of the time. When a face
was classified as ‘bad (B),’ it was attacked sixty-three percent of the time.
However, when no categorization was made, it was attacked sixty-nine
percent of the time. Results of interference effects in categorization have
been replicated by Di Nuzio and colleagues (2013).

Busemeyer et al. found that a quantum model and interference effects
in psychological processes can better handle Pr(A) found in the D-alone
condition despite the probabilities found in the C-then-D condition.
They assumed a Hamiltonian that contains the two factors of evolving
preferences over time based on payoffs and evolving beliefs over time.
Here, beliefs and preferences tend to evolve toward consistency over time.
The two-factor Hamiltonian yields a unitary transformation that entangles
beliefs and preferences over time. It is this entangled state that generates
the interference effects that lead to the violation of the law of total
probability. That a quantum model can model various aspects of cognitive
information processing including categorization and decision-making that
are problematic for classical probability theory has also been independently


The conjunction fallacy is when subjects assign a higher probability to a conjunction of events rather than to the events considered separately. For example, one may assign a certain probability that Fred is wearing a blue sweater and another probability that Fred is wearing something made in Milan, Italy, but the conjunct probability that Fred is wearing a blue sweater that is also made in Milan, Italy should be understood as being smaller than the former probabilities. It is much less likely that Fred is wearing a blue sweater made from Milan than that Fred is wearing a blue sweater or that Fred is wearing something made in Milan. The conjunction fallacy occurs when in certain scenarios, subjects assign a higher probability to the conjunction rather than to the events when considered in isolation. Committing this fallacy goes directly against classical probability theory and the law of total probability. However, Tversky and Kahneman have shown that participants generally commit this fallacy in certain cases (Tversky and Kahneman 1982, 1983). For example, participants are given the vignette of Linda who is a philosophy major interested in matters of social justice and discrimination. Subjects are asked whether it is more probable that Linda is now a bank teller or a feminist bank teller. Participants commit the conjunction fallacy by generally claiming that she is more likely a feminist bank teller. Tversky and Kahneman describe this fallacy and this violation of classical probability theory as being due to participants thinking that Linda is more representative or typical of feminist bank tellers than of just bank tellers. This is known as the representative heuristic. The background knowledge that Linda studies social justice leads subjects to understand Linda to be more typical of feminist bank tellers, but then participants commit the fallacy of equating probability order with typicality order.

Yates and Carlson have provided experimental evidence against Tversky and Kahneman’s representative account of the conjunction fallacy (Yates and Carlson 1986). They have shown that the conjunction fallacy also occurs when two events are conceptually unrelated. Consider the events Governor Blanchard will succeed in raising the Michigan state income tax and Bo Derek will win an Academy Award for the movie that she is
Currently making. Both events are unrelated such that it is difficult to see how representativeness could be the basis of the judged probability of the conjunction. In this experiment most participants made the conjunction fallacy by concluding that there is a higher probability that the conjunction will occur than that Bo Derek will win an Academy Award for the movie that she is currently making. Although the representative heuristic view fails to account for all the phenomena, there have been other more sophisticated models that have attempted to describe the various nuances of the conjunction effect, such as the classical-based simplest weighted averaging model. It posits that the conjunction probability of two events is the convex combination of the probabilities of the events when considered in isolation. However, it has problems accounting for double conjunction errors, where the probability of the conjunction of events is higher than both of the single events, and it falsely predicts that only a single conjunction error is always present, where the probability of the conjunction of events is higher than only one of the events (Franco 2009, Busemeyer, Pothos et al. 2011). Moreover, for the simplest weighted averaging model, the probability value of an item is given independent of other items with which it is paired. However, empirical evidence has shown that this independence assumption is false, and rather the interdependence of items should be taken into consideration (Busemeyer, Pothos et al. 2011).6 The averaging model also does not describe the reduction in the conjunction error when mutually exclusive events are being used.

Although the variously nuanced classical-based accounts fall short in fully describing the phenomena, several theorists have independently shown that a quantum computational model of cognitive processing for decision-making can provide a simple description for the elusive conjunction fallacy as well as for ordinary probability decision-making (Bordley and Kadane 1999, Aerts 2009, Conte, Khrennikov et al. 2009, Franco 2009, Khrennikov 2010, Yukalov and Sornette 2010, Busemeyer, Pothos et al. 2011). A quantum cognitive model is able to describe decision-making in regards to the conjunction effect that are in agreement with the well-established empirical findings. They have shown that interference effects

6 This objection also holds for the memory retrieval models.
in psychological processes that arise from the superposition of the relevant thoughts of the two events can describe why participants generally provide a higher probability for the conjunction of events. Moreover, their models appropriately describe single and double conjunction effects as well as the conjunction fallacy when two events are conceptually unrelated. Moreover, a quantum model makes an a priori prediction on the conjunction fallacy that incompatible judgments entail order effects (Busemeyer et al. 2011, Busemeyer et al. 2015), and this prediction has been empirically confirmed (Stolarz-Fantino et al. 2003).

The sure thing principle is a fundamental law of classical Bayesian probability theory. The principle states that if you prefer action $x$ over $y$ in circumstance $z$ and you prefer action $x$ over $y$ in circumstance $\sim z$, then you should prefer $x$ over $y$ when the circumstance is unknown. However, in a well-known study, this principle was tested by Tversky and Shafir in a two stage gambling experiment as well as in a prisoner’s dilemma situation (Tversky and Shafir 1992). Participants were told that they had just played a gamble where there was a fifty percent chance that they won $200 and a fifty percent chance that they lost $100. In one condition, participants were told that they won the first play, and they were then asked whether they wanted to play again. Sixty-nine percent of the subjects responded that they would like to play again. In a second condition, the same participants were told that they lost the first gamble, but fifty-nine percent of such participants replied that they would like to play again. In a third condition, the same participants were told that they did not know the outcome of the first gamble. In this situation, only thirty-six percent of the participants chose to play again. This study of what is known as the disjunction effect demonstrates the violation of the sure thing principle in decision-making as well as the law of total probability, which implies that the unknown probability should be a weighted average of the two known probabilities.

An important experiment in decision-making science is the two-person Prisoner’s Dilemma game. In this game, there are two people who originally cannot communicate with one another and are about to be sentenced for a crime they jointly committed. If they both cooperate and do not testify against each other, then they both get one year in jail. If one of them cooperates and the other defects by testifying against his partner, then the
defector will go free, and the cooperator will get three years in jail. If they both defect and testify against each other, then they both get two years in jail. There is a Nash equilibrium, where no player has an incentive to deviate from his or her chosen strategy after considering an opponent’s choice. The equilibrium is for both to defect. In our experimenter’s modified study in which participants were told in advance what their partner had chosen, in the first condition, participants were placed in the game but were told that their hypothetical partner defected. Most participants then chose to defect. In the second condition, the participants were told that their imaginary partner has chosen to cooperate. Once again, most subjects then chose to defect. In the final condition, the participants were told that you do not know what your partner has selected. In this condition, most participants chose to cooperate; thus, breaking the sure thing principle and the law of total probability. The findings of Tversky and Shafir have been replicated several times (Croson 1999, Li and Taplin 2002).

A number of theorists have independently provided quantum models that can account for the empirical findings that human beings generally violate the sure thing principle in certain circumstances (Busemeyer, Matthew et al. 2006, Aerts 2009, Conte, Khrennikov et al. 2009b, Pothos and Busemeyer 2009, Khrennikov 2010, Yukalov and Sornette 2010, Busemeyer, Pothos et al. 2011, Trueblood and Busemeyer 2011). The models account for the heuristics involved in making such a fallacy. Quantum probability theory and the existence of superpositions in quantum models can account for the surprising probabilities in the above scenarios when there is an unknown variable as well as in conditions when the variables are known. The deviant probabilities are postulated as being the result of interference effects drawn from quantum computations. However, classical probability theory fails to account for the violation of the sure thing principle since this view states that in the unknown variable circumstance, the probability for action is the average of the probabilities for the two known variable cases. Furthermore, Pothos and Busemeyer demonstrate that even when cognitive dissonance is taken into the calculations, classical probability theory still does not account for the empirical phenomena while quantum probability theory can (2009). As previously stated, the ability of a quantum cognitive processing model to account for the above empirical phenomena has been shown independently
by numerous theorists.

Although we obviously do not have the space to discuss every study pertaining to the quantum mechanical mind in relation to decision-making,\(^7\) interference effects at the psychological level have been demonstrated to be able to account for ordinary acts of decision-making but also for a variety of decision-making acts that are problematic for classical probability theory, such as with attitude and human preference questions (Aerts and Aerts 1994, Khrennikov 2004), cognitive judgments (Aerts and Aerts 1994, Khrennikov 1999, Mura 2005, Yukalov and Sornette 2008), measurement of expected utility (Mura 2009), conceptual combination (Gabora and Aerts 2002, Aerts 2009, Bruza, Kitto et al. 2009),\(^8\) and cooperation in prisoner’s dilemma games (Eisert, Wilkens et al. 1999, Piotrowski and Sladkowski 2003). Furthermore, a quantum formalism in light of empirical studies can accurately describe numerous other fallacies, heuristics, and biases committed and used, respectively, in decision-making that are problematic for classical probability theory, such as unpacking effects (Busemeyer, Pothos et al. 2011), the inverse and base rate fallacy (Franco 2008b, Villejoubert 2002), framing effects (Mogiliansky, Zamir et al. 2006, Khrennikov 2007, Lambert-Mogiliansky, Zamir et al. 2009), Ellsberg’s paradox (Mura 2009), order effects (Lambert-Mogiliansky, Zamir et al. 2009, Busemeyer, Pothos et al. 2011, Trueblood and Busemeyer 2011), the effects of risk and ambiguity (Mura 2005, Franco 2007), and the gambler’s and hot hand fallacies (Franco 2008a).

Quantum models can also account for perceptual decisions (Atmanspacher, Filk et al. 2004, Conte et al. 2009), cognitive dissonance (Lambert-Mogiliansky, Zamir et al. 2009), learning, memory, information-processing, and approach-avoidance behavior (Ivancevic and Aidman 2007), and how a subject alternates between perceiving the statements in a liar paradox as being true and false (Aerts, Broekaert et al. 2004). Moreover, studies have shown that in many cases of cognition, quantum computations are faster

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\(^7\) For a good summary of the many various studies in quantum cognition, see Busemeyer and Bruza 2012.

\(^8\) Although conceptual combination is not a kind of decision-making *per se*, it is involved in decision-making in that we need to combine concepts in order to construct complete thoughts or decisions.

The descriptive success of a quantum formalism for ordinary acts of decision-making as well as for many fallacies, heuristics, and biases that still have not been appropriately described by the classical model lends strong support that human decision-making is correctly described by quantum cognitive processes. Quantum cognitive models for decision-making surprisingly are able to describe numerous phenomena, some of which are listed as paradoxes from the perspective of classical probability theory. The general theory of relativity is able to describe many phenomena just as with Newtonian physics, but was empirically confirmed as having greater descriptive adequacy based on only three kinds of crucial phenomena. However, as we can see, quantum cognitive models of decision-making in light of classical cognitive models can do much better than this. Quantum cognitive models of decision-making are able to appropriately describe similar phenomena as compared to classical models, especially for ordinary decision-making acts. However, the novel successful descriptions made by quantum models for numerous different kinds of psychological fallacies, heuristics, biases, and other decision-making phenomena firmly places quantum theory over classical theory in this regard. Unlike the general theory of relativity’s established superiority over classical physics, there are much more than three kinds of distinguishing and empirically tested differences between quantum and classical cognitive models in which replicated empirical evidence establishes descriptive superiority of quantum theory over classical probability theory. Although no works in quantum cognition have synthesized the above findings to draw a general conclusion about decision-making, given the strong replicated support for a quantum formalism that covers a wide diverse range of decision-making phenomena and by putting together the puzzle pieces of various works and studies by quantum cognitive scientists, we now have sufficient evidence to inductively infer the general conclusion that the processes underlying psychological decision-making are best described by quantum processes.
3. Objections

One may object that even though averaging models based on Markov decision-making cannot fit empirical results on the conjunction fallacy and other psychological phenomena, this is not a problem. There potentially can be other models with classical-based decision rules that can generate violations of the law of total probability and the sure-thing principle. The decision-making processes can involve a lot of strategic reasoning, and they do not have to be guided by a Markov chain. Sufficiently complex classical reasoning possibly can yield probabilities that resemble those from quantum theories. It may be possible to construct classical computations that accurately simulate quantum computations.

In response to this counter, while this suggestion may be possible, the fact of the matter is that classical models have had problems with and have failed to sufficiently describe the numerously listed above relevant psychological phenomena in a coherent way. Classical models have been shown to be able to describe certain psychological phenomena as well as quantum models can. However, quantum models have an overall greater success in describing decision-making as compared to classical models when taking into account all the various decision-making phenomena. As discussed, classical theorists have attempted to make more complex models and have explored various possible routes for describing normal cases of a type of decision-making, such as probability decision-making, as well as using the model to describe aberrations within that type of decision-making, such as the conjunction fallacy. The trick is to construct a coherent classical model that can describe normal and abnormal decision-making, not just construct some ad hoc incoherent conglomeration of various computational processes (Busemeyer & Bruza 2012, Blutner & beim Graben 2015, Bruza et al. 2015). For instance, the quantum description of the conjunction fallacy is simplistically axiomatic coming from its coherent set of principles rather than being a patchwork description that appends an isolated ad hoc heuristic computational process in order to try and describe the conjunction

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9 There are classical models for contextuality (Dzhafarov 2014, 2015) on par with quantum ones (Busemeyer & Bruza 2012). However, contextuality does not involve decision-making per se, so I do not go into further detail on this here.
fallacy as well as sewing in different isolated ad hoc components in order to attempt to address different phenomena. Yet, classical theorists have failed in constructing such a successful coherent model, a model where descriptions inevitably stem from the coherent set of basic principles, for a plethora of various types of decision-making, while quantum theorists have not.

On top of this, classical models that employ heuristics (Simon 1957) and ones that adopt a rational approach, where people make decisions in a rational way based on Baye’s rules and the expected utility rule (Savage 1954), have been around for over sixty years. It is striking that in less than twenty years, quantum models have been able to provide appropriate descriptions of all the previously discussed decision-making phenomena that create difficulties for classical models. This provides further significant support for quantum models over classical ones.

Even though there is the possibility that classical models might be shown to be successful in the future, all things considered, quantum models of decision-making are superior to classical ones. Although stronger arguments may be made for quantum models, at minimum, based on an inductive inference, the conclusion can be drawn that given all the evidence and timeframes, there is a strong likelihood that the quantum approach for decision-making is the correct descriptive theory as compared to classical probability theories. It is significantly more likely than not. Hence, we are justified in believing that the quantum approach is superior to the classical one, and we would be unjustified in believing the reverse. As previously stated, quantum models have been experimentally shown on numerous replicated occasions to be able to describe psychological phenomena and processes that classical models have simply failed to coherently describe. Due to this fact, quantum models overall have a greater ability to describe the phenomena. Also, recall that there are fundamental and empirically tractable different descriptions made by these two probability theories, where the quantum view has appropriately described the many above cases of decision-making. Quantum probability theory justifiably has replaced classical probability theory in microphysics as it can describe phenomena that cause problems for classical probability theory, *inter alia*. Likewise, in terms of providing descriptions of cognitive decision-making processes,
quantum probability models justifiably should replace classical probability models in the realm of psychological decision-making as they coherently can describe phenomena that cause significant difficulties for classical probability theory.

Second, one may object that at the moment there is no unified quantum cognitive model, but rather, there are several different models that describe the various decision-making phenomena. However, for theoretical simplicity, there should be only one unified quantum model in cognitive science for decision-making. This point is correct that as of yet in the current literature, there is not a single consensus quantum cognitive decision-making model that has been arrived upon by quantum cognitive scientists. Take for example the phenomenon of borderline vagueness. The notions of ‘a tall man’ and ‘not a tall man’ do not have exact boundaries as to who falls within their extensions. Moreover, there are borderline cases, where for certain individuals it is unclear whether the predicate ‘a tall man’ applies to them or not. While classical fuzzy set logic and probability theory have failed to describe the phenomenon of borderline vagueness, different quantum models of decision-making have been put forth to successfully describe such cases (Blutner et al. 2013, Sozzo 2014). In this instance, we have two separate quantum models from Blutner et al. versus Sozzo that agree that human decision-making in regards to borderline vagueness can be adequately described by quantum cognitive processes, where borderline vagueness is an effect of a quantum interference phenomenon between the notions of ‘a tall man’ and ‘not a tall man.’ Nevertheless, both models have significant differences from each other. For example, Sozzo’s model is set in Fock space rather than in a Hilbert space. Moreover, following the lead of the likes of Freud, James, and Piaget, Sozzo’s model adopts a two-layered form in human thought, logical and conceptual, as playing a basic role in the formation of borderline contradictions, while Blutner’s model does not. Also, Blutner et al.’s model accepts that the de Morgan’s rules are satisfied while Sozzo’s model allows for the possibility that de Morgan’s laws are violated. Despite both being quantum models, there are sharp structural and conceptual differences between them. Further work is required to adjudicate between these two views.

Despite this general problem of a lack of a unified quantum view,
the general framework of the various quantum cognitive models still commonly rely on such ideas as superposition, quantum interference, and other common fundamental concepts of quantum physics. While there is not a single quantum model as of yet that has been demonstrated to describe and unify all the decision-making cognitive phenomena, this possibility is certainly not ruled out, and such a model may be developed in the future. Furthermore, individual quantum models have shown the power to unify over a number of phenomena that from classical probability theory, requires several different models. For example, the leading classical theory that attempts to describe cognitive dissonance and the leading classical theory that tries to describe framing effects, respectively, are very different theories. As we can see, this objection at hand also applies to classical-based models. In this particular case of discussion, Ariane Lambert-Mogiliansky and company provide a single quantum model that can describe both phenomena of cognitive dissonance and framing effects (2009). The leading non-quantum theories that attempt to describe Allais’ and Ellsberg’s paradoxes are different models, but La Mura’s quantum model can successfully account for both paradoxes (2009). A single quantum model can also unify the conjunction effect, disjunction effect, order effects, and categorization judgments (Bruza et al. 2015), whereas a single classical model up to this point has failed in this endeavor. Keeping in mind that it has been shown that decision-making at the psychological level can best be described by the class of quantum models as compared to the class of classical ones, this empirically confirmed capacity for quantum models to unify where classical probability theory has not lends some theoretical simplicity to the given formalisms. Furthermore, this ability to unify where the classical has not as well as the fact that quantum models fundamentally rely on the same basic concepts of quantum physics provides the potential for the development of a consensus quantum model in the future that can unify all the decision-making cognitive phenomena.

As a final worry, if decision-making for the mind is quantum mechanical, does the brain also have to be quantum mechanical? Yet, recall that whether the brain is quantum mechanical or not is a highly controversial subject in which it may be the case that there is not enough empirical evidence as of yet to establish that neurobiological events are significantly based on
quantum effects. To note, quantum cognitive scientists generally have not addressed this objection or question in regard to the brain.

Nevertheless, we may bypass this objection based on a functionalist philosophy of mind for intentional non-phenomenal decision-making psychological states. On this standard view for non-phenomenal mental states, the relevant mental states are higher order functional properties that play a causal role within the causal network of the mind; a mind that receives environmental inputs, has mental state to mental state causal interactions, and produces behavior. As it is generally conceived, once mental states are functionally defined, they are constituted by whatever lower level physical states realize the causal role. On a general understanding of this view, although the mental supervenes upon the physical, mental states are not identical to physical states since mental properties are higher level properties that may be multiply realized in various ways at the neurobiological level depending on the person or even life form that is in question.

The brain does not have to operate based on quantum effects since the brain is not identical to the psychological. Since the psychological and neurobiological deal with two distinct kinds of properties that lie at different levels, it is perfectly feasible to have the decision-making mind be essentially based on quantum computations and processes while the brain is not. This would be like an ordinary classical computer that has been installed with a physics software program that can be used to solve problems in quantum physics by relying on quantum computational processes, but the hardware of the computer is classical. The quantum software can be multiply realized by various arrangements of physical states.

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10 Functionalism is the widely accepted view for non-phenomenal mental states in the philosophy of mind. Due to the explanatory power of functionalism, David Chalmers famously labels decision-making mental states and processes as being part of the “easy” problem of consciousness (1996). In the 1960’s, functionalism replaced the identity theory of the mind, where mental states are identical to brain states, due to numerous objections, such as Putnam’s famous multiple realizability contention.

11 Blutner & beim Graben claim that functional states can be identified with regions in the state space of the neural network of the brain which multiply realize mental states (2015). Different partitions of this neural space could be conceived as complementary, thus leading to emergent quantum-like properties.
computer hardware. Although the mental supervenes and depends upon the physical in order to be realized, on this picture of the mind and brain, decision-making at the psychological level is still quantum mechanical as it is the quantum computations and cognitive processes that are responsible for why you reached one decision rather than another while the lower level brain processes are classical. Quantum processes are responsible for decision-making. Of course, if it does turn out that the brain is quantum mechanical as well as the mind, then I still get my conclusion that decision-making is quantum mechanical. Hence, regardless of whether the brain is classical or quantum, this appears to not matter for my conclusion on decision-making.

4. Conclusion

I have contributed to the literature by synthesizing various studies in quantum cognition that focus on specific kinds of decision-making to overall contend the general position that psychological decision-making is best described by a quantum formalism. Furthermore, I at times have provided novel defenses of this view from numerous objections. Coupled with the many foundational arguments as to why a quantum model of decision-making is necessary and explanatorily adequate, a paradigm shift concerning the foundations of decision-making science is warranted. Future energies and research should be wholly shifted and devoted to working on various aspects of quantum theories of decision-making just as was the case for the theory of quantum mechanics in the micro world after it first was confirmed over classical physics many years ago. For instance, reallocated energies were devoted to expanding the scope of applicability of quantum mechanics throughout the micro world and unifying microphysics.

A central task for future research for quantum modeling is to arrive upon a unified quantum theory of decision-making. Furthermore, this unified framework should also be able to account for acts of cognition that may not include decision-making per se. While decision-making has been the sole focus and scope of my paper and a large emphasis in quantum cognition has focused on various kinds of decision-making, there is relatively limited work by quantum theorists on other aspects of cognition such as memory,
comprehension, learning, language production, and attention. Unification is one sign of a mature discipline towards which the field of quantum cognition should strive.

Another possible future line of research is to examine what implication quantum models of decision-making have on the free will debate. Philosophers (Kane 1998), neuroscientists (Libet et al. 1983), and psychologists (Wegner 2002) who apply the mind sciences to the free will debate largely do not discuss quantum models of decision-making in their writings. As deterministic computational processes are largely presumed in the scientific study of free will and decision-making, quantum models of decision-making may shed a different indeterministic light on the subject matter.

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